

Challenge

NATIONAL CIPHER

A free codebreaking competition
for UK Schools & Colleges
Over £3,000 in prizes

UNIVERSITY OF Southampton

School of Mathematics

Competition starts 1st October 2009
Register online from 7th September

The Byron

The race to build the world's first computer is threatened by an international conspiracy. Only one person can save the Analytical Engine from falling into the wrong hands

YOU!

Join forces with mathematician Ada Lovelace, the world's first computer scientist, to crack the Byron Enigma ...

Enigma



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www.cipher.maths.soton.ac.uk

Teachers' notes and lesson plans

email: cipher@soton.ac.uk

Welcome to the 8th National Cipher Challenge

www.cipher.maths.soton.ac.uk

About the Challenge

Welcome to the National Cipher Challenge, a nationwide online codebreaking competition designed to get young people excited about mathematics and computing. The competition will run from October 1st 2009 to January 2nd 2010 and is a great extension activity or a fantastic maths club project which can be tackled by pupils in teams or on their own. Based on a brand new historical adventure involving Ada Lovelace, Charles Babbage and other heroes of Victorian science the story will unfold in a series of short encrypted messages for the participants to crack, using all their skill and cunning. Together with the lesson plans and notes contained in this Teachers' Pack, the competition is a great introduction to the skills of a codebreaker, introducing the competitors to the thrill of discovery experienced by professional mathematicians and computer scientists in their work.

Over the next few months you will be joining Ada Lovelace and Charles Babbage as they endeavour to save the world's first computing engines, the Difference Engine and the Analytic Engine from falling into the hands of arch criminals and enemies of the British Empire. You will also need to help them break the Byron Enigma, a subtle puzzle left by Byron as a challenge for his daughter, Ada. It will be your job to help Lovelace and Babbage in their efforts to decipher cryptic messages passed between their enemies, and these messages will get harder and harder to crack as the adventure unfolds. You will also need to decipher the letters between Ada and Charles.

Challenges will be set on the web-site www.cipher.maths.soton.ac.uk in the Challenges section, and will come in two parts. Part A will consist of letters, notes etc. sent by Ada Lovelace and her friends and colleagues. You can expect these messages to be fairly lightly encrypted, at least at first, although in the latter stages of the competition security will be tightened and you will find the part A ciphers harder to crack. In particular Challenge 8A will consist of the Byron Enigma, and this will be quite difficult, but if you have followed the competition you should by that stage be an arch-cryptographer and it should be within your reach. Part B consists of intercepted messages between the conspirators as they plot to steal the computers. As these messages are intended in many cases to be sent to agents of foreign powers, and since the conspirators have so much to hide, these messages will often be much more securely encrypted than the part A challenges, though you may find that the part A messages (and the previous part B messages themselves) contain clues to the decryption of the part B texts, so it is worth deciphering the part A challenge even if your main interest is the part B competition..

If you get stuck on a Challenge don't give up, sometimes a good night's rest is all you need. Other times you need more practical help and you can turn to the website for clues. You might find them posted (by us) as comments, on the bulletin board, though we ask you not to post hints of your own without checking them with us first as this

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will spoil the Challenge for others. (Anyone posting solutions or links to solutions may be barred from the site and disqualified from the competition.)

The Challenges will be published at 3.15pm on a Thursday, according to the schedule below. You can submit your solution any time after the challenge has been published and before the deadline at one minute to midnight the day before the next Challenge is published. Solutions to each Challenge will be published after the deadline, so if you can't crack a particular Challenge don't give up. After you submit you can check for feedback on your solution by logging in at this page (we delay the feedback as an incentive to get it right first time, so you will lose points if you rely on it instead of correcting your own mistakes!). If you have submitted a solution with a mistake in it you can then submit again (though watch out for rule 20 below). For each Challenge you will receive a score based on your speed and accuracy, and we will use these scores to compile an Honours Board for each Challenge part A and for each part B. These lists will be published so you can see how you are getting on. The part B Honours Lists will be compiled into the overall Championship Leader Board which will be used to determine the winners of the competition. Entries for each challenge, which may be from individuals or from teams, should be submitted using the submissions web-form. Be careful to follow the instructions on the form. Sorry, but failure to adhere to all the instructions could result in an entry being deemed invalid. The detailed rules are given online, so make sure you read them well.

www.cipher.maths.soton.ac.uk/rules

What do you need to do?

To take part you will need to register for the competition on our registration page:

www.cipher.maths.soton.ac.uk/registration

This will be open from September 7th, and you will need to provide the following information:

- **Username:** This will be the name you use to log on to the site to post comments, and also to submit your entries, check feedback and to print your certificate. Choose something memorable. You can share this info among the team, but do not let anyone else have it as they can use it to pose as you!
- **Password:** Again this is for logging on. Choose it carefully and keep it secret to the team.
- **Email address:** This will be used to confirm your registration so it must be an active account you can check. We can only associate one email account to each team, so it might be an idea to set up an account (with gmail or some other free provider) that the whole team can use for the competition. If we need to contact you this is how we will do it so add the account cipher@soton.ac.uk to the account addressbook to avoid sending our emails to your junk mail bin, make sure the account is not too full, and check it regularly.
- **Team name:** This will be used on the downloadable certificates and the leader boards to identify you. Try to choose something interesting but tasteful. (We reserve the right to refuse offensive names!)
- **Team members for the web:** Enter your names as you wish them to appear on the leader board. If you wish to remain anonymous online then leave this blank or use a nickname that does not identify you.
- **Team members for the certificates:** These are your names as you wish them to appear on the downloadable certificates and will not be used publically without your permission.
- **Number in team:** While we could be clever and write a script to try and compute the number in the team from the info about team member names this is not very accurate and far too much work, so please tell us how many of you are in the team so we get some idea of how many are taking part!
- **Teacher contact:** Give the name of a teacher we can write to if we need to check anything. You should get their permission first! We don't usually do this unless you win a prize. If you are homeschooled give us a parent or carer's name here and write homeschooled in the schoolname field. You will still need to give us contact details in the address fields below.
- **School name:** Again we will use this on the certificates and the leader boards so spell it right, and if it is a common name (like King Edward's) add a location (King Edward's, Southampton) to identify it.
- **School address:** If you win a prize this is how we will get it to you, so it is important that you get this right. We collect the postcode separately so we can easily map the participants to give us some idea of the spread of entries.
- **School postcode:** See above.

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This year we are trying something new!

Every member of a team can have their own account and link it to the team account so they can submit and read feedback using their own username. To do this the team captain must set up his/her account and invite the other team members to join the team using the group tab in the account profile. The team members will then be able to accept the invite by going to the same place in their own account profile. Individuals can leave a team at any time, but cannot be thrown out by anyone else, even the captain, so choose your teammates wisely. Oh, and the captain can only leave once everyone else has! And team submissions (from anyone) belong to the team; you can't claim them as yours if you leave. This might all sound a bit heavy handed, but it isn't meant to, it's just that this is an experiment and we want to keep it simple (or as simple as we can!)

Frequently Asked Questions

How many can enter?

Teams of any size and composition may enter, and a school can enter as many teams as it wishes. Teams can be run from one or several individual accounts – see above!

Does everyone in a team need their own account?

No, but everyone can have one! The team is principally associated with one account holder (the team captain), but team captains can invite others to join the team at any point including when it is set up. You don't need to have an individual account to be associated with a team, but you do if you want to be able to post solutions for it or to log in to see feedback and get certificates.

What does it cost to take part?

At the moment we are lucky to have several generous sponsors, and there is no charge for taking part in this year's competition.

When does the competition start?

Registration opens on September 7th. The first part of the competition will be published at 3.15 on Thursday October 1st. There is no need to rush to download it as you have one or two days in which to submit to achieve full speed marks. Often the website is overloaded for the first half an hour or so, and it probably pays to wait out the rush.

When will the Challenges be published?

*At or shortly after 3.15pm on each of the following Thursdays:
1st, 8th, 15th, 22nd October; 5th, 19th November; 3rd, 17th December.*

How long do we have to complete each challenge?

The deadline for each Challenge is 11.59pm on the day before the next Challenge is published, though the number of points you will score depends to a certain extent on how early you submit your best attempt.

What age group is it aimed at?

Principally this is an extension activity for older pupils, but the early stages of the competition are aimed at a wide audience and there is something for everyone. We have had bright year 6 pupils do well on early rounds and this prepares them for further achievement in future competitions. Many pupils return year after year as they try to improve on the number of stages they can successfully complete, and the staged certificates give everyone an incentive to keep going as long as possible. There are also some small prizes awarded at random to a few participants each week as an added incentive.

Is this appropriate for a Math Club activity?

Certainly. The entire math club could enter as a team or you could divide up into smaller groups and use the math club meetings to discuss techniques and strategies.

Can pupils enter on their own?

Yes, we get many solo entrants and teachers do not have to be involved, but we do ask for the name of a teacher contact for prize administration.

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Do team members have to all come from the same school?

No, in the past we have had several teams made up of members of different schools and colleges, and this is perfectly acceptable. However we do ask for the name of at least one of the schools and a teacher contact for prize administration, and, for now at least, the team captain's school will get all the credit!

Where can I find out more?

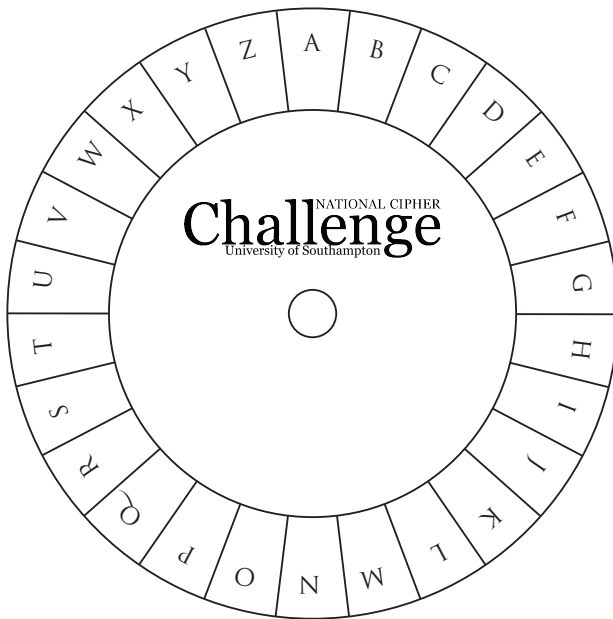
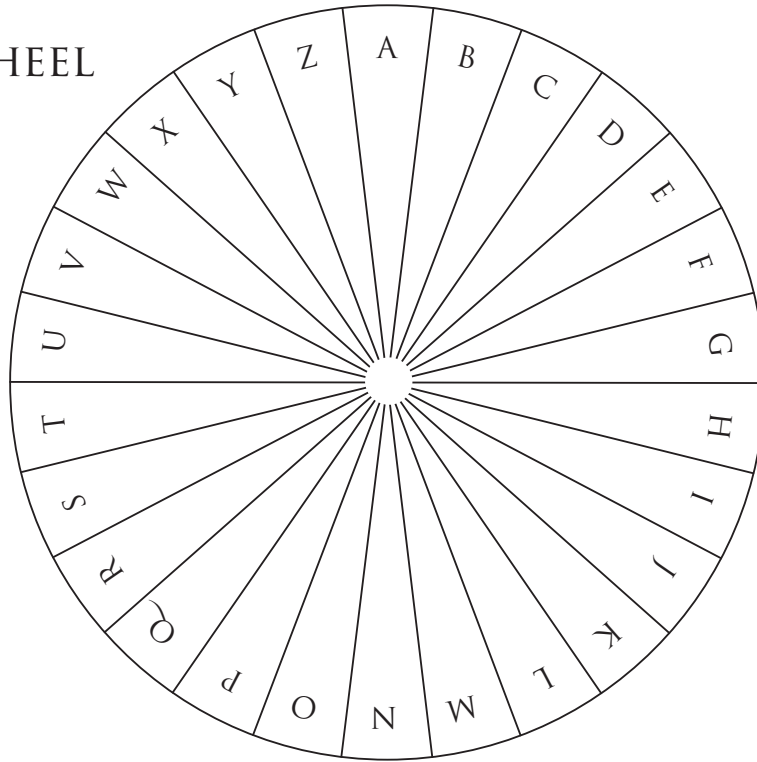
The rules page and the news and info pages give more information, and it is worth taking part in the forums, which we will monitor

www.cipher.maths.soton.ac.uk/rules

<http://www.cipher.maths.soton.ac.uk/news-and-info>

<http://www.cipher.maths.soton.ac.uk/discussion-boards>

CIPHER WHEEL



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On substitution ciphers

Graham A. Niblo
University of Southampton National Cipher Challenge 2008

These notes form a brief introduction to substitution ciphers, to accompany the lesson plans provided with the University of Southampton National Cipher Challenge, 2005. We would like to thank Hugh Evans of Sholing City Technology College for his assistance in the design of these teaching materials.

Caesar shift ciphers

The easiest method of enciphering a text message is to replace each character by another using a fixed rule, so for example every letter a may be replaced by D, and every letter b by the letter E and so on.

Applying this rule to the previous paragraph produces the text

WKH HDVLHVW PHWKRQ RI HQFLSKHULQJ D WHAW PHVVDJH LV WR
UHSODFH HDFK FKDUDFWHU EB DQRWKHU XVLQJ D ILAHG UXOH, VR
IRU HADPSOH HYHUB OHWWHU D PDB EH UHSODFHG EB G, DQG
HYHUB OHWWHU E EB WKH OHWWHU H DQG VR RQ.

(Note the convention that ciphertext is written in capital letters, while plaintext is usually lowercase.)

Such a cipher is known as a shift cipher since the letters of the alphabet are shifted round by a fixed amount, and as a Caesar shift since such ciphers were used by Julius Caesar. To decode a Caesar shift it is enough to work out the amount of shift, which can be done, for example, by discovering which character has replaced the letter e. In the example above we might guess that the three-letter word starting the sentence is the and therefore that the letter e has been replaced by H. A quick check shows that the Caesar shift by 3 does indeed encode the word the as WKH, and it is easy to complete the decryption.

In fact there are only 26 Caesar shift ciphers (and one of them does nothing to the text) so it is not too hard to decipher the text by brute force. We can try each of the shifts in turn on the first word of the cipher text until we discover the correct shift. This process can be simplified by using a cipher wheel, a simple mechanical device which allows one to generate each of the Caesar shift ciphers, and to encode or decode messages using it. At the back of this pack you will find a sheet, which can be photocopied onto thin card in order to make a cipher wheel. Cut out the two discs, and fasten through their centres with a paper fastener to make the wheel. Use the convention that you read plaintext on the outer rim of the wheel and cipher text from the smaller wheel.

Keyword substitution ciphers

To increase the difficulty of deciphering the text we need a richer family of ciphers. A good example is furnished by the keyword cipher. In this we design an encryption table by choosing a keyword or phrase, which is used to jumble the alphabet as follows:

Write down the phrase, with no spaces between the letters, and omitting any repeated character. So if the phrase is “The Simpsons” we write down THESIMPON. Now we continue to go round the alphabet until every letter appears exactly once, and write the list under the standard alphabet:

```
abcdefghijklmnopqrstuvwxy  
z  
THESIMPONQRUVWXYZABCDFGJKL
```

Of course if the key phrase is carefully chosen (for example “The quick brown fox jumps over the lazy dog”) we may not need to complete the list as above, but such a choice is not necessary. The number of such ciphers is $26!$, or approximately 10^{27} , and brute force cannot be used to attack the problem. However an attack is possible.

Consider the text

```
VEP HYXHLVHTP MO AWFJYFLT H RFNEPS HJNEHAPV FL VEFU ZHC  
FU VEHV FV FU PHUC VM KPKMSFUP VEP IPCZMSY MS IPCNESHUP,  
HLY EPLRP VEP RFNEPS HJNEHAPV. VEFU FU FKNMSVHLV, APRHWUP  
FO VEP UPLYPS EHU VM IPPN VEP RFNEPS HJNEHAPV ML H NFPRP  
MO NHNPS, VEP PLPKC RHL RHNWSP VEP NHNPS, YFURMXPS VEP  
IPC, HLY SPHY HLC RMKKWLFRRHFVMLU VEHV EHP APPL PLRSCNVPY  
ZFEV FV. EMZPXPS FO VEP IPC RHL AP RMKKFVVPY VM KPKMSC FV  
FU JPUU JFIPJC VM OHJJ FLVM PLPKC EHLIU.
```

As before we notice that the first word has three letters and, since it occurs several times, may well be the word *the*. This gives a strong hint that the letter *e* is enciphered as the letter *P* in the keyphrase cipher. Of course other three letter words are possible, *e.g.*, *and* or *but*. Nonetheless a quick check shows us that the letter *P* is the most common letter in the enciphered text, just as *e* is the most common letter in English so it is reasonable to assume that the correct decryption translates *P* to *e*. This also suggests that *V* stands for *t* and *E* for *h*, allowing us to begin to decipher the text. We will use the convention that uppercase letters denote enciphered letters and lowercase denotes plaintext characters:

```
the HYXHLtHTe MO AWFJYFLT H RFNheS HJNhhAet FL thFU ZHC  
FU thHt Ft FU eHUC tM KeKMSFue the IeCZMSY MS IeCNhSHUe,  
HLY heLRe the RFNheS HJNhhAet. thFU FU FKNMStHLt, AeRHWUe  
FO the UeLYeS hHU tM IeeN the RFNheS HJNhhAet ML H NFeRe  
MO NHNeS, the eLeKC RHL RHntWSe the NHNeS, YFURMXeS the  
IeC, HLY SeHY HLC RMKKWLFRRhtFMLU thHt hHXe AeeL eLRSCNteY
```

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ZFth Ft. hMZeXeS FO the IeC
RHL Ae RMKKFtteY tM KeKMSC Ft
FU JeUU JFIeJC tM OHJJ FLtM eLeKC hHLYU.

Reading carefully we see the single letter word H, and the four letter word thHt in the first line, and guess that H enciphers the letter a. Making that replacement we get:

the aYXaLtaTe MO AWFJYFLT a RfNheS aJNhaAet FL thFU ZaC
FU that Ft FU eaUC tM KeKMSFUe the IeCZMSY MS IeCNhSaUe,
aLY heLRe the RfNheS aJNhaAet. thFU FU FKNMStalt, AeRaWUe
FO the UeLYeS haU tM Ieen the RfNheS aJNhaAet ML a NFeRe
MO NaNeS, the eLeKC RaL RaNtWSe the NaNeS, YFURMXeS the
IeC, aLY SeaY aLC RMKKWLFratFMLU that haXe AeEL eLRSCNteY
ZFth Ft. hMZeXeS FO the IeC RaL Ae RMKKFtteY tM KeKMSC Ft
FU JeUU JFIeJC tM OaJJ FLtM eLeKC haLYU.

Now the two 2 letter words Ft FU are probably "it is" meaning that F enciphers "i" and U enciphers "s". Hence we get:

the aYXaLtaTe MO AWiJYiLT a RiNheS aJNhaAet iL this ZaC
is that it is easC tM KeKMSise the IeCZMSY MS IeCNhSase,
aLY heLRe the RiNheS aJNhaAet. this is iKNMStalt, AeRaWse
io the seLYeS has tM Ieen the RiNheS aJNhaAet ML a NieRe
MO NaNeS, the eLeKC RaL RaNtWSe teh NaNeS, YisRMXeS the
IeC, aLY SeaY aLC RMKKWLiRatiMLs that haXe AeEL eLRSCNteY
Zith it. hMZeXeS io the IeC RaL Ae RMKKitteY tM KeKMSC it
is Jess JiIeJC tM OaJJ iLtM eLeKC haLYs.

Continuing with appropriate guesses (haXe = have, easC = easy and so on) we decipher the text to get the following extract from Simon Singh's excellent history of codes and ciphers, *The Code Book*:

"The advantage of building a cipher alphabet in this way is that it is easy to memorise the keyword or keyphrase, and hence the cipher alphabet. This is important, because if the sender has to keep the cipher alphabet on a piece of paper, the enemy can capture the paper, discover the key, and read any communications that have been encrypted with it. However if the key can be committed to memory it is less likely to fall into enemy hands."

Frequency analysis

A more methodical attack is frequency analysis. One counts the number of occurrences of each character in the cipher text and compares it with an expected frequency for the standard English alphabet. In the cipher text above a character count gives us the following table of occurrences:

a	b	c	d	e	f	g	h	i	j	k	l	m
7	0	12	0	26	27	0	32	6	9	11	20	18
n	o	p	q	r	s	t	u	v	w	x	y	z
16	5	55	0	14	17	2	17	35	4	4	11	4

Compare this to a table of expected frequencies, taken from Simon Singh's "The Code Book":

a	b	c	d	e	f	g	h	i	j	k	l	m
8.2	1.5	2.8	4.3	12.7	2.2	2.0	6.1	7.0	0.2	0.8	4.0	2.4
n	o	p	q	r	s	t	u	v	w	x	y	z
6.7	7.5	1.9	0.1	6.0	6.3	9.1	2.8	1.0	2.4	0.2	2.0	0.1

Using this and information about common one, two and three letter words we have enough to begin to tackle the cipher.

Disguising the word structure

The chink in the armour of our ciphers so far has been the preservation of word structure. This allows one to spot common words. In order to avoid such weakness cryptographers usually remove punctuation and block the characters together in groups of four or five, so our previous cipher text looks like

```
VEPHY XHLVH TPMOA WFJYF LTHRF NEPSH JNEHA PVFLV EFUZH
CFUVE HVFVF UPHUC VMKPK MSFUP VEPIP CZMSY MSIPC NESHU
PHLYE PLRPV EPRFN EPSHJ NEHAP VVEFU FUFKN MSVHL VAPRH
WUPFO VEPUP LYPSE HUVMI PPNVE PRFNE PSHJN EHAPV MLHNF
PRPMO NHNPS VEPPL PKCRH LRHNV WSPVE PNHNP SYFUR MXPSV
EPIPC HLYSP HYHLC RMKKW LFRHV FMLUV EHVEH XPAPP LPLRS
CNVPY ZFVEF VEMZP XPSFO VEPIP CRHLA PRMKK FVVPY VMKPK
MSCFV FUJPU UJFIP JCVMO HJJFL VMPLP KCEHL YU
```

Usually the length of the text groups doesn't matter, however, in analysing a Vigenère cipher (see below) a carelessly chosen block length may make the length of the keyword more apparent, since it can reveal the repetitions more easily.

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To attack cipher text that has been grouped in this way we have to work with letters not words. To do so we use the frequency analysis described above, together with a little judgement (or luck!). The process can be hard, but wars have been won or lost on the back of it, and so have fortunes.

“It was hard going, but Jericho didn’t mind. He was taking action, that was the point. It was the same as code-breaking. However hopeless the situation, the rule was always to do *something*. No cryptogram, Alan Turing used to say, was ever solved by simply staring at it.” From *Enigma*, by Robert Harris.

Affine shift ciphers

Despite the advantages for an agent in using keyword substitution ciphers most modern ciphers are automated and rely on a mathematical encryption algorithm. Indeed the Caesar shift cipher can be viewed as just such a cipher:

We start by encoding each letter by its numerical position in the alphabet:

a	b	c	d	e	f	g	h	i	j	k	l	m
1	2	3	4	5	6	7	8	9	10	11	12	13
n	o	p	q	r	s	t	u	v	w	x	y	z
14	15	16	17	18	19	20	21	22	23	24	25	26

Next we shift the alphabet by adding 3 to each position:

a	b	c	d	e	f	g	h	i	j	k	l	m
4	5	6	7	8	9	10	11	12	13	14	15	16
n	o	p	q	r	s	t	u	v	w	x	y	z
17	18	19	20	21	22	23	24	25	26	1	2	3

Of course $24+3 = 27 \neq 1$, but here we are carrying out modular arithmetic, familiar as clock arithmetic, so that when we reach 26 we continue from 1.

Finally we replace the numbers with the letters they stand for:

a	b	c	d	e	f	g	h	i	j	k	l	m
d	e	f	g	h	i	j	k	l	m	n	o	p
n	o	p	q	r	s	t	u	v	w	x	y	z
q	r	s	t	u	v	w	x	y	z	a	b	c

This recovers the cipher table constructed in lesson plan 1 for the Caesar shift by 3.

There is a convenient shorthand for the Caesar shift by n , given by $x \rightarrow x+n$. It is confusing since here we are using x to stand for the position of a letter, and n to stand for the shift amount, *i.e.*, x and n are each one of the values 1 ... 26. It is clear that since the shift is defined by the integer n there are only 26 Caesar shift ciphers.

There is a bigger class of shift ciphers which can be written in these terms known as the affine shift ciphers, and they exploit the fact that we can multiply as well as add integers in modular arithmetic. It is slightly complicated to set up formally but rather easy to do in practice so we will work through an example.

The affine shift $x \rightarrow 3x+5$

We start as before with the position table, but this time instead of replacing a position x with the number $x+3$ we will replace it by the number $3x+5$, where this number is interpreted appropriately. So for example $2 \rightarrow 3 \cdot 2+5 = 11$, while $8 \rightarrow 3 \cdot 8+5 = 29$ which is interpreted as 3 ($29=26+3$). Whenever the result of the computation is larger than 26 we keep subtracting 26 until it becomes smaller. More formally we compute $3x+5$ and then take the remainder after division by 26. This yields the table:

a	b	c	d	e	f	g	h	i	j	k	l	m
8	11	14	17	20	23	26	3	6	9	12	15	18
n	o	p	q	r	s	t	u	v	w	x	y	z
21	24	1	4	7	10	13	16	19	22	25	2	5

And from this we recover the encryption table as given on the handout for lesson 3:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
H	K	N	Q	T	W	Z	C	F	I	L	O	R	U	X	A	D	G	J	M	P	S	V	Y	B	E

The affine shift ciphers can also be written in a shorthand form $x \rightarrow ax+b$ and the Caesar shift ciphers are special cases of the affine shift ciphers with $a=1$.

Now notice that in both the Caesar shift $x \rightarrow x+3$ and the affine shift $x \rightarrow 3x+5$ the letter y is enciphered as B, since $25+3 = 28 = 26+2$, and $3 \cdot 25+5 = 80 = 3 \cdot 26+2$. It follows that two different affine shift ciphers can encrypt a letter in the same way, so it is no longer sufficient to discover the letter substituting for e in order to decipher the message. Since there are two degrees of freedom in our choice of cipher we might hope that deciphering two letters is sufficient, and it is, since, if we know two values of the expression $ax+b$ we can solve the two corresponding simultaneous equations to find a and b .

We may be more familiar with this exercise when solving the equations over the real numbers, but the same method works for modular arithmetic, with the caveat that in general we cannot divide. This caveat has an interpretation in cryptography. In order for the rule $x \rightarrow ax+b$ to define a cipher it had better be the case that each of the numbers $1 \dots 26$ appears exactly once in the list of numbers $ax+b$ as x ranges from 1 to 26. If we choose a carelessly (so that we can't divide by $a \pmod{26}$) this might not be the case.

For example the rule $x \rightarrow 2x$ tries to encipher both m and z as Z, since $2 \cdot 13=26$ and $2 \cdot 26 = 52$ both of which are equal to 26 modulo 26. Such an encryption cannot easily

be deciphered since the recipient of the message is unable to determine whether the sender intended Z to be read as m or z .

From a mathematician's point of view the enciphering rule defines a function from the alphabet to itself, and this needs an inverse if the cipher is to be decipherable in a deterministic way. In other words the number theory function $x \rightarrow ax+b$ needs to have an inverse in mod 26 arithmetic. It is a fact from elementary number theory that it will have such an inverse if and only if a is coprime to 26, that is, their only common divisor is 1.

There are 12 numbers less than 26 and coprime to it (those odd numbers not divisible by 13) so we have 12 possible choices of the number a , and 26 choices for the number b , yielding 312 affine shift ciphers. This makes a brute force attack, without frequency analysis, less practical than the much simpler situation for Caesar shift ciphers.

Polyalphabetic ciphers

The main weakness allowing us to tackle a substitution cipher is the irregularity in the distribution of letters in English text. Other languages demonstrate similar (though language specific) irregularities and you can find frequency tables for them on the web.

In order to remove this weakness from a cipher it is necessary to disguise the frequencies of letters in the plaintext and the easiest way to do this is by using a polyalphabetic cipher. In such a cipher each plaintext letter may be encoded in more than one way so that, for example, the letter e may be enciphered as both X and G within the ciphertext. One problem with this approach is that if X and G both encode for e we don't have enough letters left to encode the other 25 letters. One elegant solution to this problem is the famous French cipher known as the Vigenère cipher.

In a Vigenère cipher ANY letter might be encoded by any other; a given Vigenère cipher uses a subset of the 26 possible Caesar shift ciphers. Of course for a genuine recipient to have any hope of deciphering the message there has to be a way to determine for each cipher character which of the shifts has been used. The answer to this tricky problem is to choose a sequence of them known to both parties but to no-one else.

So the two parties might agree to use shifts of 22, 9, 7, 5, 14, 5 18, and 5 in that order and to continue repeating the pattern for the entire text: 22, 9, 7, 5, 14, 5 18, 5, 22, 9, 7, 5, 14, 5 18, 5, 22, 9 *etc.*

In order to decode the cipher text the recipient shifts the first cipher character back by 22, the second back by 9 and so on to recover the cipher text. Of course the question remains how one can memorise the correct sequence, but here we borrow an idea from the keyword cipher. The shift numbers 1, ..., 26 are taken to stand for the alphabet a, \dots, z , and then the pattern 22, 9, 7, 5, 14, 5 18, 5 spells the word *vigenere*.

To set up a Vigenère cipher the two parties agree in advance to use the shift pattern encoded by some agreed keyword or phrase; in our previous Golden Jubilee Cipher challenge we used a Vigenère cipher based on the keyword GOLD, so characters were shifted in turn by 7, 15, 12, 4. Such a cipher is very hard to crack.

The method we recommend is due to Babbage and Kasiski who independently discovered it, and is based on the regularity of the repetition. An analysis of repeated strings of letters is used to try to determine the length of the keyword, and once this is done a standard frequency analysis is applied to each part of the ciphertext encoded by a single cipher. A very good account of Babbage-Kasiski deciphering can be read in chapter 2 of Simon Singh's *The Code Book*.

On transposition ciphers



Sometimes when you carry out a frequency analysis you will find that each letter occurs with about the same frequency as you would expect in natural English text (or whichever language you are studying). This is a broad hint that the text is not enciphered using a substitution cipher, but rather by an anagram or transposition cipher, also known as an anagram cipher. In such a cipher the letters of the message are not replaced by substitutes, but rather jumbled using some rule which allows them to be untangled again to decipher the message.

Example

We will encipher the text:

The quick brown fox jumps over the lazy dog

We start with a keyword. Suppose our keyword is BAD. We write it at the head of a table with three columns, then enter the ciphertext in the boxes below. The last, empty, box is padded with an X.

B	A	D
t	h	e
q	u	i
c	k	b
r	o	w
n	f	o
x	j	u
m	p	s
o	v	e
r	t	h
e	l	a
z	y	d
o	g	x

Next we rearrange the columns so that the letters in the keyword are now in alphabetic order

A	B	D
H	T	E
U	Q	I
K	C	B
O	R	W
F	N	O
J	X	U
P	M	S
V	O	E
T	R	H
L	E	A
Y	Z	D
G	O	X

Giving a ciphertext of

HTEUQIKCBORWFNOJXUPMSVOETRHLEAYZDGOX

If the keyword contains repeated letters then we delete them as we would if it were the keyword for a substitution cipher before constructing the grid. Hence if the keyword was TOFFEE we would use a grid of width 4 with header TOFE and we would rearrange the grid so that its header appeared as EFOT to encipher the message.

How do we tackle such a cipher?

Clearly the length of the keyword is quite crucial. You should be able to guess this from the length of the ciphertext, which will be a multiple of it. So in our example the ciphertext has length 36 which has factors 2,3,4,6,9 and so on. Hence we could try laying out the text in grids of width 2,3,4,6,9 respectively (a keyword of length 12 or more is unlikely) and examining the rows.

Of course a grid of width 2 would leave us just switching alternate letters so we probably don't need to lay it out that way to check it. Having checked and dismissed the idea of a keyword of length 2 the first grid we try looks like the second grid above. Having got to this point the best hope for a quick solution is to find a crib. If there is a word you think ought to appear in the cipher text then you could try looking for anagrams of that word. This is made difficult by the fact that in splitting the text into blocks (blocks of three in the example), If your crib word does not take up an entire block then even the characters from the crib that do appear will be jumbled with other nearby characters, so you need a reasonably long crib. On the other hand if it is too long only part of the word will appear in that block so you are looking for anagrams of parts of the crib.

In our example if we knew, for some reason, that the text was likely to contain the word "jumps" we could look for anagrams of "jum", "ump", "mps. Looking carefully you should see the anagram PMS in the text and we might guess that the first and

second columns have been transposed while the third has remained fixed. Checking this we find have cracked the cipher.

Things are harder with longer keywords but the principle remains the same. Things get tougher if the plaintext is not in our own language, since it is harder to say what makes sense. Of course even in this case it may be that part of the message is in your language and the rest in another. In this case you might hope to crack the ciphertext corresponding to your native language, and apply the knowledge that gives you about the cipher to write down a decrypt of the entire message, even when the text is unfamiliar.

Other (subtle) cribs: In English the letters q and u occur together so if they are separated either you are not looking at English text or they should be brought back together by undoing the anagram.

Numbers often represent dates, so for example the letters/numbers 2, 1, s, t in proximity might represent 21st, while 2,1,t,h might represent 12th.



Cryptography Lesson Plan 1

Class: Cracking the Caesar shift ciphers.

Resources:

- Leaflet “On substitution ciphers”.
- Two handouts each with a plaintext and a cipher table
- Teachers’ solutions for the handouts.
- One OHP slide with cipher text to crack, and partial decrypt and solution.

Starter: (10 minutes approximately) Uses handouts for Groups A and B
Encryption exercise – split the class into groups A and B. Give each group the enclosed text to encipher using the given code. Encourage accuracy AND secrecy! Answers enclosed with handouts.

Main activity: (40 minutes approx) Uses OHP

- *Introduce the idea of a substitution cipher in general and the Caesar shift in particular.*
- *Suggest trial and error as a deciphering technique.*
- *Work through a very simple Caesar shift (by 3).*
- *Split the class again, swap over the ciphertexts from the starter exercise and get them to tackle them.*

Plenary (approx 10 minutes)

Discuss how to make the code harder to crack using a rule that is harder to determine, but remark on the need for an easy to remember rule (stressed agents must remember it and can’t write it down!) Mention “keyword” substitution.

Handout for lesson 1.



GROUP A

Code: Caesar shift by 2

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B

Plaintext

There were plenty of schools in the world, but they were all run either by the various churches or the Guilds. Miss Butts objected to churches on logical grounds and deplored the fact that the only Guilds that considered girls worth educating were the Thieves and the Seamstresses. It was a big and dangerous world out there and a girl could do worse than face it with a sound knowledge of geometry and astronomy under her bodice.

From "Soul Music" by Terry Pratchett.

Handout for lesson 1.

GROUP B

Code: Caesar shift by 4

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D

Plaintext

The four houses are called Gryffindor, Hufflepuff, Ravenclaw and Slytherin. Each house has its own noble history and each has produced outstanding witches and wizards. While you are at Hogwarts, your triumphs will earn your house points, while any rule-breaking will lose house points. At the end of the year the house with the most points is awarded the House Cup, a great honour. I hope each of you will be a credit to whichever house becomes yours.

From “Harry Potter and the Philosopher’s Stone” by J.K. Rowling.

Teachers' solutions to encryption challenge



Ciphertext A

VJGTG YGTG RNGPVA QH UEJQQNU KP VJG YQTNF, DWV VJGA YGTG
CNN TWP GKVJGT DA VJG XCTKQWU EJWTEJGU QT VJG IWKNFU.
OKUU DWVVU QDLGEVGF VQ EJWTEJGU QP NQIKECN ITQWPFU CPF
FGRNQTGF VJG HCEV VJCV VJG QPNA IWKNFU VJCV EQPUKFGTGF
IKTNU YQTVJ GFWECVKPI YGTG VJG VJKGXGU CPF VJG
UGCOUVTGUUGU. KV YCU C DKI CPF FCPIGTQWU YQTNF QWV VJGTG
CPF C IKTN EQWNF FQ YQTUG VJCP HCEG KV YKVJ C UQWPF
MPQYNGFIG QH IGQOGVTA CPF CUVTQPQOA WPFGT JGT DQFKEG.

Ciphertext B

XLI JSYV LSYWIW EVI GEPPIH KVCJJMRHSV, LYJJPITYJJ,
VEZIRGPEA ERH WPCXLIVMR. IEGL LSYWI LEW MXW SAR RSFPI
LMWXSVC ERH IEGL LEW TVSHYGIH SYXWXERHMRK AMXGLIW ERH
AMDEVHW. ALMPI CSY EVI EX LSKAEVXW, CSYV XVMYQTLW AMPP
IEVR CSYV LSYWI TSMRXW, ALMPI ERC VYPI-FVIEOMRK AMPP PSWI
LSYWI TSMRXW. EX XLI IRH SJ XLI CIEV XLI LSYWI AMXL XLI
QSWX TSMRXW MW EAEVHIH XLI LSYWI GYT, E KVIEX LRSYV. M
LSTI IEGL SJ CSY AMPP FI E GVIHMX XS ALMGLIZIV LSYWI
FIGSQIW CSYVW.

OHP Slide for lesson 1

Ciphertext

*WKH HDVLHVW PHWKRGI HQFLSKHULQJ D WHAW PHVVDJH LV WR
UHSODFH HDFK FKDUDFWHU EB DQRWKHU XVLQJ D ILAHG UXOH, VR
IRU HADPSOH HYHUB OHWWHU D PDB EH UHSODFHG EB G, DQG HYHUB
OHWWHU E EB WKH OHWWHU H DQG VR RQ.*

Partial decrypt: Guess that the first word is “the” so that t is enciphered as W, h as K and e as H. This suggests a shift by 3:

the eDVLeVt PethRG RI eQFLSheULQJ D
teAt PeVVDJe LV tR UeSODFe eDFh
FhDUDFteU EB DQRtheU XVLQJ D ILAeG
UXOe, VR IRU eADPSOe eYeUB OetteU D
PDB Ee UeSODFeG EB G, DQG eYeUB OetteU
E EB the OetteU e DQG VR RQ.

The word teAt could be tent, test or text, with text fitting with the shift by 3; the word OetteU which occurs twice, would decipher to “letter” confirming our guess.

Code: Caesar shift by 3

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

Plaintext

The easiest method of enciphering a text message is to replace each character by another using a fixed rule, so for example every letter a may be replaced by d, and every letter b by the letter e and so on.



Cryptography Lesson Plan 2

Class: Cracking keyword substitution ciphers – emphasises letter frequency analysis and team work.

Resources:

- Leaflet “On substitution ciphers”.
- OHP 1 containing ciphertext
- OHP 2 Containing expected frequency table and incomplete actual frequencies.
- Handout summarising details of deciphering technique.
- OHP 3 With further thoughts on disguising the text.

Starter: (10 minutes approximately) (Uses OHP 1)

Split the class into teams and get them to count the letter frequencies in the ciphertext. Emphasise the need for speed and accuracy. Maybe set the scene as a race against time.

Main activity: (30 minutes approx) (Uses OHP 1 and OHP 2 and handout)

- *Introduce the idea of a keyword cipher to make encryption more secure and more memorable (see “On substitution ciphers”).*
- *Discuss the hunt for common words and letters and introduce frequency analysis – show a table of common frequencies and check it against the examples in lesson 1.*
- *Discuss the speed improvements given by parallel processing of the text. Split into 26 teams to do a frequency analysis of the given ciphertext on OHP 1. {It may be worth remarking that standard computer attacks on ciphers use this idea of parallel processing to speed up the attack.}*
- *Whole class session to construct frequency table, compare with expected frequencies (computed from percentages) and identify the letters “e” and “t”.*

Plenary (20 minutes approx) (Uses OHP 2 and, time permitting OHP 3)

Draw together the intelligence gained by the groups and crack the cipher together. (You may wish to give out the handout summarising the technique after completing the exercise.)

If time permits (OHP 3):

- *Discuss how to make the code harder to crack by disguising the letter groups.*
- *Remark that the frequency table can mislead for non-standard or foreign language texts! Examine the extract from the book “A Void” by Georges Perec.*

OHP Slide 1 for lesson 2



Ciphertext

VEP HYXHLVHTP MO AWFJYFLT H RFNEPS HJNEHAPV
FL VEFU ZHC FU VEHV FV FU PHUC VM KPKMSFUP
VEP IPCZMSY MS IPCNESHUP, HLY EPLRP VEP
RFNEPS HJNEHAPV. VEFU FU FKNMSVHLV, APRHWUP
FO VEP UPLYPS EHU VM IPPN VEP RFNEPS
HJNEHAPV ML H NFPRP MO NHNPS, VEP PLPKC RHL
RHNWSP VEP NHNPS, YFURMXPS VEP IPC, HLY
SPHY HLC RMKKWLF RHVFMLU VEHV EHXP APPL
PLRSCNVPY ZFVE FV. EMZPXPS FO VEP IPC RHL
AP RMKKFVVPY VM KPKMSC FV FU JPUU JFIPJC VM
OHJJ FLVM PLPKC EHLJU.

OHP Slide 2 for lesson 2

Occurrences table

a	b	c	d	e	f	g	h	i	j	k	l	m
n	o	p	q	r	s	t	u	v	w	x	y	z

Expected Frequency table

a	b	c	d	e	f	g	h	i	j	k	l	m
8.2	1.5	2.8	4.3	12.7	2.2	2.0	6.1	7.0	0.2	0.8	4.0	2.4
n	o	p	q	r	s	t	u	v	w	x	y	z
6.7	7.5	1.9	0.1	6.0	6.3	9.1	2.8	1.0	2.4	0.2	2.0	0.1

This table was taken from “The Code Book” by Simon Singh, and gives expected frequencies as a percentage. To accurately compare it to the actual frequencies above you should compute the actual frequencies as percentages.

Actual Frequencies as percentages

a	b	c	d	e	f	g	h	i	j	k	l	m
n	o	p	q	r	s	t	u	v	w	x	y	z



Handout for lesson 2

STAGE 1 – P is the commonest letter in the ciphertext so could stand for e - maybe the first word is the:

the HYXHLtHTe MO AWFJYFLT H RFNheS
HJNnHAet FL thFU ZHC FU thHt Ft FU eHUC
tM KeKMSFUe the IeCZMSY MS IeCNhSHUe,
HLY heLRe the RFNheS HJNnHAet. thFU FU
FKNMStHLt, AerHWUe FO the UeLYeS hHU tM
Ieen the RFNheS HJNnHAet ML H NFeRe MO
NHNeS, the eLeKC RHL RHntWSe the NHNeS, YFURMXeS the IeC,
HLY SeHY HLC RMKKWLFrHtFMLU thHt hHXe AeEL eLRSCNteY ZFth
Ft. hmZeXeS FO the IeC RHL Ae RMKKFtteY tM KeKMSC Ft FU
JeUU JFIeJC tM OHJJ FLtM eLeKC hHLYU.

STAGE 2 We see the single letter word H, and the four letter word thHt in the first line - guess that H encodes the letter a.

the aYXALtATe MO AWFJYFLT a RFNheS aJNhaAet FL thFU ZaC
FU that Ft FU eaUC tM KeKMSFUe the IeCZMSY MS IeCNhSaUe,
aLY heLRe the RFNheS aJNhaAet. thFU FU FKNMStALt, AerAWUe
FO the UeLYeS haU tM Ieen the RFNheS aJNhaAet ML a NFeRe
MO NaNeS, the eLeKC RaL RaNtWSe the NaNeS, YFURMXeS the
IeC, aLY SeaY aLC RMKKWLFrAtFMLU that haXe AeEL eLRSCNteY
ZFth Ft. hmZeXeS FO the IeC RaL Ae RMKKFtteY tM KeKMSC Ft
FU JeUU JFIeJC tM OaJJ FLtM eLeKC haLYU.

STAGE 3 The two 2 letter words Ft FU are probably it is meaning that F encodes i and U encodes s:

the aYXALtATe MO AWiJYiLT a RiNheS aJNhaAet iL this ZaC
is that it is easC tM KeKMSise the IeCZMSY MS IeCNhSase,
aLY heLRe the RiNheS aJNhaAet. this is iKNMStALt, AerAWse
iO the seLYeS has tM Ieen the RiNheS aJNhaAet ML a NieRe
MO NaNeS, the eLeKC RaL RaNtWSe teh NaNeS, YisRMXeS the
IeC, aLY SeaY aLC RMKKWLiRatiMLs that haXe AeEL eLRSCNteY
Zith it. hmZeXeS iO the IeC RaL Ae RMKKitteY tM KeKMSC it
is Jess JiIeJC tM OaJJ iLtM eLeKC haLYs.

STAGE 4: haXe = have, easC = easy and so on - we get the following extract from Simon Singh's excellent history of codes and ciphers, *The Code Book*:

"The advantage of building a cipher alphabet in this way is that it is easy to memorise the keyword or keyphrase, and hence the cipher alphabet. This is important, because if the sender has to keep the cipher alphabet on a piece of paper, the enemy can capture the paper, discover the key, and read any communications that have been encrypted with it. However if the key can be committed to memory it is less likely to fall into enemy hands."

Obscuring a substitution cipher

1. We can disguise the word structure by regrouping the letters into blocks:

VEPHY XHLVH TPMOA WFJYF LTHRF NEPSH JNEHA
PVFLV EFUZH CFUVE HVFVF UPHUC VMKPK MSFUP
VEPIP CZMSY MSIPC NESHU PHLYE PLRPV EPRFN
EPSHJ NEHAP VVEFU FUFKN MSVHL VAPRH WUPFO
VEPUP LYPSE HUVMI PPNVE PRFNE PSHJN EHAPV
MLHNF PRPMO NHNPS VEPPL PKCRH LRHNV WSPVE
PNHNP SYFUR MXPSV EIPIC HLYSP HYHLC RMKKW
LFRHV FMLUV EHVEH XPAPP LPLRS CNVPY ZFVEF
VEMZP XPSFO VEPIP CRHLA PRMCK FVVPY VMKPK
MSCFV FUJPU UJFIP JCVMO HJJFL VMPLP KCEHL YU

2. We can distort the frequency table – this text was adapted for last years cipher challenge!

Augustus, who has had a bad night, sits up blinking and purblind. Oh what was that word (is his thought) that ran through my brain all night, that idiotic word that, hard as I'd try to pin it down, was always just an inch or two out of my grasp - fowl or foul or Vow or Voyal? - a word which, by association, brought into play an incongruous mass and magma of nouns, idioms, slogans and sayings, a confusing, amorphous outpouring which I sought in vain to control or turn off but which wound around my mind a whirlwind of a cord, a whiplash of a cord, a cord that would split again and again, would knit again and again, of words without communication or any possibility of combination, words without pronunciation, signification or transcription but out of which, notwithstanding, was brought forth a flux, a continuous, compact and lucid flow: an intuition, a vacillating frisson of illumination as if caught in a flash of lightning or in a mist abruptly rising to unshroud an obvious sign - but a sign, alas, that would last an instant only to vanish for good.

From "A Void" by Gilbert Adair. The letter "e" does not appear even once in the book!

Cryptography Lesson Plan 3

Class: Affine shift ciphers – emphasises clock arithmetic and gives more practice at frequency analysis.



Resources:

- Leaflet “On substitution ciphers”.
- OHP 1, giving partial encryption table for the $3x+5$ affine shift cipher together with teachers' solution.
- OHP 2-4, with cipher text to crack, method and solution.

Starter: (10 minutes approximately) Uses handout

Complete the encryption table on the OHP (the affine shift cipher $x \rightarrow 3x+5$ is discussed in the teachers' notes).

Encourage them to try to spot the pattern and guess the rule which should be concealed.

Main activity: (40 minutes approx) Uses OHP

- *Introduce the class of affine shift ciphers mentioning “clock arithmetic” mod 26*
- *Show them that the cipher table arises from the affine shift $x \rightarrow 3x + 5$.*
- *Discuss the fact that you only need to know the value of two letters to deduce the affine shift (solving two simultaneous equations mod 26).*
- *Use frequency analysis and modular arithmetic to decipher an affine shifted text together or in groups.*

Plenary (approx 10 minutes)

Discuss generalisations to modular arithmetic mod n .

OHP slide 1 for lesson 3.

Spot the pattern?

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
H	K	N					C																		

$$x \rightarrow 3x + 5$$

1	2	3	4	5	6	7	8	9	10	11	12	13
8	11	14	17	20	23	26	3	16	9	12	15	18
14	15	16	17	18	19	20	21	22	23	24	25	26
21	24	1	4	7	10	13	16	19	22	25	2	5

Encryption table

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
H	K	N	Q	T	W	Z	C	F	I	L	O	R	U	X	A	D	G	J	M	P	S	V	Y	B	E

OHP Slide 2 for lesson 3



Ciphertext

LMYFU BKUUS DDYFA XWCLA OLPSF AOLMJ FASDS
 NSFGJ FAOEL SOMYT DJLAX EMHJM BFMIB JUMIS
 HFSUL AXUBA FKJAM XLSKF FKXWS DJLSO FGBJM
 WFKIU OLFMX MTMWA OKTTG JLSXL SKFFK XWSDJ
 LSI ZG TSXWJ LJLSX LSUMF JSDJL SIZGH FSQYS
 XOGLS DMMDT SDMXJ LSBAT SMHBK BSFLS BFMCT
 SDKFM YXDJL SLYJM ZTANA MYUXM CJMCL MCKUT
 MMEAX WKJLA IKXDC LMCKU XJJLA UCKUC LKJAJ
 LKDZS SXTAE SHMFJ SXAXJ SFIAX KZTSI MXJLU
 TKUJG SKFXM CMXDS FLSLK DWMXS IKDJL SOLMF
 YUTAX SMHIS KXAXW TSUUT SJJSF UDKXO SDZSH
 MFSLA USGSU ZYJLJ SGCSF SXMJI SKXAX WTSUU
 JLSGC SFSTM KDSDC AJLJL SIMUJ NAJKT ISKXA
 XWAIK WAXKZ TSAHM XTGLS OMYTD HAXDA JZYJC
 LSFSC KUJLS BKJJS FXCLS FSCUJ JLSBK JJSFX
 CLSFS CKUJL SBKJJ SFXHF MISXA WIKZG FMZSF
 JLKFF AU

Occurrences table:

A	B	C	D	E	F	G	H	I	J	K	L	M
34	12	19	23	4	41	12	10	16	50	38	46	42
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
3	11	1	1	0	78	22	28	0	13	41	11	11

Use frequency analysis to guess that **S** enciphers for **e**, and **J** for **t**.

This tells us that for an affine shift cipher

$$x \rightarrow ax + b$$

$$a.5 + b = 19 \quad (e \rightarrow S)$$

$$a.20 + b = 10 \quad (t \rightarrow J)$$

Solving mod 26 we see that $15.a = -9 \pmod{26}$. Now 7.15 is congruent to 1 mod 26 since $7.15 = 105 = 104 + 1 = 4.26 + 1$. It follows that $7.15.a = 7.-9$, or a is congruent to -63 .

Now $-63 = -52 - 11$, so a is congruent to -11 , or equivalently to $15 \pmod{26}$. Hence $a = 15$. Now from $a.5 + b = 19$ we get $75 + b$ is congruent to 19, or b is congruent to $-56 \pmod{26}$.

Since $-56 = -2.26 - 4$, b is congruent to $-4 \pmod{26}$ so $b = 22$.

To check this $20.a + b = 300 + 22 = 322 = 12.26 + 10$, so $a.20 + b = 10$ as required. So the affine shift is $x \rightarrow 15x+22$ and the decrypt is given by the inverse function $y \rightarrow 7(y-22)$

[It might look strange but “dividing by 15” is the same as multiplying by 7 in mod 26 arithmetic.]

Equivalently the decryption is achieved by the affine shift $y \rightarrow 7y+2$.

Encryption table:

a	b	c	d	e	f	g	h	i	j	k	l	m
I	P	W	D	K	R	Y	F	M	T	A	H	O
n	o	p	q	r	s	t	u	v	w	x	y	z
V	C	J	Q	X	E	L	S	Z	G	N	U	B

OHP Slide 4 for lesson 3



Decrypt

hours passe dduri ngwhi chjer ichtot riede
veryt rickh ecoul dthin kofto promp tsome
fresh inspi ratio nhear range dthec rypto
grams chron ologi cally thenh earra ngedt
hemby lengt hthen hesor tedth embyf reque
ncyhe doodl edont hepil eofpa perhe prowl
edaro undth ehuto blivi ousno wtowh owasl
ookin gathi mandw howas ntthi swasw hatit
hadbe enlik efort enint ermin ablem onths
lasty earno wonde rheha dgone madth echor
uslin eofme aning lessl etter sdanc edbef
orehi seyes butth eywer enotm eanin gless
theyw erelo adedw ithth emost vital meani
ngima ginab leifo nlyhe could findi tbutw
herew asthe patte rnwhe rewas thepa ttern
where wasth epatt ernfr omeni gmaby rober
tharr is